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On Optimal Routing for Commercial Formation Flight

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This paper explores an extension to a geometric approach of finding optimal routes for commercial formation flight. An adaption of the Breguet range equation, alongside specific aircraft characteristics, is used to represent realistic aircraft and underlying changes in weight as fuel is burnt off. Weighting schemes, for both nominal and differential rates of fuel burn, are introduced and compared. Finally a method for finding wind-optimal routes in a formation flight paradigm is developed in order to assess the effectiveness of a geometric estimate for the formation pair assignment problem. Using the geometric method to allocate formation pairs is shown to offer good performance for solutions with a significant reduction in computation time against all possible wind-optimal formation routes.

I. Introduction

Commercial aviation is constantly looking for ways to cope with predicted increases to future demand[1] whilst simultaneously trying to mitigate the resulting impact on the environment. This paper explores the possibility of flying in formation, as an alternative to the way commercial flight operates today, in an attempt to optimize current routes and decrease overall fuel burn.

One of the immediate benefits of formation flight, over other proposed fuel saving methods,[2–4] is the relatively minimal change to the current infrastructure. The majority of today's commercial airliners can fundamentally observe a reduction in drag from formation flight.[5] Although the possibility of designing new aircraft in the future to take advantage of the aerodynamic benefits of this scenario would be a long term goal, in the short term it would not be a necessity.

Studies into areas of biomimicry such as geese flying in a 'V' formation[6,7] have always interested scientists, while the military have long flown in formation for communicative and defensive purposes. More recent studies assessing the aerodynamic possibility of flying in close proximity in order to reduce drag[8] coupled with real time flight tests[9,10] shows promise that flying in formation can reduce fuel burn and in turn improve performance factors such as range and speed.

While some studies show a positive trade off between deviating routes, in order to fly in formation and the reduction in drag it produces,[11–15] few have tackled the substantial fleet-assignment problem when routing for formation flight. The massively combinatorial nature of this task means that in order to assess sizeable problems a smart approach is needed. Both centralized and decentralized approaches are explored in Ref. 16, wherein a small case study for the two-aircraft problem is covered. The incorporation of 'proposal-marriage' type algorithm explores the idea of joining formation in an ad-hoc fashion. Route optimization studied in Ref. 5, along with a case study, shows significant cost saving potential, while using a more in-depth optimizer, solutions obtained retain many of the restrictions imposed by today's infrastructure.

Although there is a clear interest in harnessing formation flight to improve aircraft performance,[5,11–16] little work has been done on the large scale allocation problem. That is, given a set of possible solo flights, how to go about assigning them to particular formation 'fleets'. The problem in question is highly combinatorial and therefore as the number of flights or size of the fleets increase the possible ways of joining them together grows dramatically. This paper proposes a time-free possible solution method through the use of a Fermat-Toricelli approach which precedes the route assignment problem. Section II begins by first reviewing this fast geometric approach to finding time-free optimal routes with few constraints (previously introduced by the authors in Ref. 17). Then sections III and IV make an extension to this framework to include an increasing level of detail arising from aircraft specific performance factors and differential rates of fuel burn.

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An adaption of the well known Breguet range equation[18] is used to gain approximations to how the rate at which fuel burns changes during a flight. Finally section V presents a simple method for optimal routing in the presence of wind, in order to assess the effectiveness of the wind-free geometric solution as an initial estimate for the global wind-optimal formation pairings.

II. Review of a Geometric Solution Method

The overtly combinatorial nature of the problem requires a clever approach in how it is modelled. In order to realistically compute the rapidly increasing number of possible route combinations an equally rapid method is required. The problem of formation routing can be abstracted to a simpler geometric approach through the use of an extension of the classic weighted Fermat-Torricelli point problem.[19] A full analytic methodology for finding optimal rendezvous points, for aircraft to meet and fly in formation, to try to minimize a given cost function was first developed by the authors in Ref. 17. The following reviews some of the key points used to determine geometrically optimal routes.

The original 'weight-free' Fermat-Torricelli problem[19] involves finding a single point P for a given triangle ABC which minimizes the sum of the distances $AP + BP + CP$. There have been numerous solutions since it was first proposed in the 17^{th} century, some involving mechanics and differentials while most rely on a variety of mathematical properties based on the geometric dualities between triangles and circles.

In order to appropriately model the formation flight problem a scalar cost weighting is assigned to each arc to represent each section of a formation flight. Given a triangle ABC with vertices A and B representing two distinct departure airports and the third vertex C a common destination, the minimizing point P therefore represents where the two aircraft, originally flying A to C and B to C, would rendezvous to fly in formation. Under this scenario the arcs AP and BP would be the solo stage of flight, rendezvousing at P, the arc PC would be the formation stage. In addition the solo arcs are assigned weights (in line with their relative cost for flying solo) w_A and w_B, while the formation section's weight, w_f, is some proportion $\alpha \in (0, 1]$ (representing the fuel saving by flying in formation) of the combined solo weights, that is, $w_f = \alpha\,(w_A + w_B)$. Therefore for the triangle ABC we seek a weighted vectorial equilibrium about the point P so that

$$w_A \frac{\overrightarrow{PA}}{||\overrightarrow{PA}||} + w_B \frac{\overrightarrow{PB}}{||\overrightarrow{PB}||} + w_f \frac{\overrightarrow{PC}}{||\overrightarrow{PC}||} = 0. \tag{1}$$

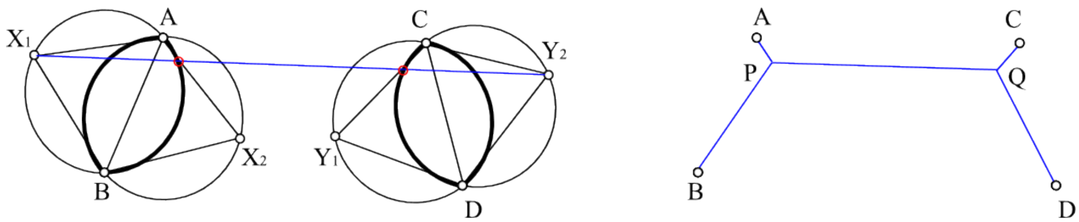
An application of the law of cosines to the three vectors in (1), results in expressions (as in equation (2)) for θ_A, θ_B and θ_f for the respective intersection angles $\angle BPC$, $\angle APC$ and $\angle APB$, based only on the input of the three scalar weight values w_A, w_B and w_f.[20]

$$\theta_A = \cos^{-1}\left(\frac{-w_B^2 - w_f^2 + w_A^2}{2w_B w_f}\right), \theta_B = \cos^{-1}\left(\frac{-w_A^2 - w_f^2 + w_B^2}{2w_A w_f}\right), \theta_f = \cos^{-1}\left(\frac{-w_A^2 - w_B^2 + w_f^2}{2w_A w_B}\right). \tag{2}$$

Importantly this means given three scalar weights, representing the cost value along each section of the flight, the angles at which these routes are required to meet in order to be optimal can now be calculated. In doing so the departure and destination nodes are decoupled and therefore any two fixed points A and B and the angle at which the trajectories meet, θ_f, defines a loci of possible formation points. In turn two corresponding inscribed circles with A and B on their perimeter can be constructed. Each circle is comprised of two arcs, the first contains, on its boundary, all the points P such that $\angle APB = \theta_f$ (*i.e.* they meet at the angle required by equation (2)), the other, a single analytically determined point (called a back vertex) ensures the remaining two angle conditions are met. A line connecting this back vertex to a destination vertex intersects the loci of possible points once and is the desired P. Furthermore this method of decoupling enables routes with both distinct departure and destination nodes to also be considered. Rather than joining the back vertex to a single destination node it is instead joined to a second back vertex, the one which corresponds to the loci of possible breakaway points at the destination (Figure 1). The authors' previous work in Ref. 17 also describes an analogous method for creating formations larger than two (The work of this paper only examines routing for formations of size two).

Lastly it is necessary to note here that the planar methodology outlined translates nicely to a spherical surface (the earth). Straight lines become planes, which intersect the sphere along great circle paths, while inscribed circles become inscribed spheres intersecting the sphere to form small circles.

Figure 1. Join and Break Points for Two Distinct Routes



(a) Trajectory Between Back Vertices

(b) Optimal Route

III. Realistic aircraft weightings

Eurocontrol's Base of Aircraft Data[21] (BADA) outlines detailed operational and performance factors. The data contains aircraft performance models for a wide range of common aircraft types and is broken down into three files. The Operations Performance File (OPF) contains all the thrust, drag and fuel coefficients to be used together with information on weights, speeds, maximum altitude, et cetera. The Airlines Procedure File (APF) defines a default operational climb, cruise and descent speed schedule that is likely to be used by an airline. The Performance Table File (PTF) presents the nominal performance of the aircraft model in the form of a look-up table. Having access to this data allows a more accurate incorporation of aspects such as speed, flight levels, climb, cruise and descent profiles into the model.

By only looking to create formations during cruise, the climb and descent section of the flight can be considered 'sunk costs', as they are carried out irrespective of any formation. Assuming a constant nominal fuel burn at a particular flight level then the objective is to minimize the total mass of fuel burnt over the cruise section of the flights (fuel has an inherent cost associated with it). With this metric in place the previously outlined weighting system can be altered to relate directly to how much fuel a particular aircraft (nominally) burns per kilometre of distance flown at cruise. The BADA enables direct calculation of these values for differing aircraft, resulting in a more realistic weighting scheme. A proportional formation weighting factor α (taken to be 0.9 for formations of size two from table 1) is still used i.e. $w_f = \alpha(w_A + w_B)$. Table 1 demonstrates possible values of α from estimates in Ref. 12–15 for varying fleet sizes.

Size of fleet (n)	1	2	3	4	5	6	7
Weight per fleet member ($w_{\alpha,n}$)	1	0.9	0.85	0.82	0.8	0.785	0.775

Table 1. Estimated Proportional Formation Weighting Factor For Fleets of Size n

Further still, BADA determines realistic rates of climb and descent for any given aircraft, enabling realistic constraints on radial distances aircraft must be away from airports before they can rendezvous with or break away from other aircraft.[17]

IV. Differential Fuel Burn Model

The above outlined framework allows aircraft-specific weightings based on a nominal mass of fuel burnt per km of distance flown at cruise. This nominal amount, however, does not incorporate the fact that as an aircraft flies it burns fuel, so decreases in weight, resulting in a lower rate of fuel burn at later stages of a flight. For example, if one flight travels 1000 km before it meets another, which has flown only 300 km, then a nominal ratio of weights may not accurately reflect this. Therefore the method needs to be able to move from a notion of a constant nominal fuel burn to one which changes with respect to distanced flown.

Using a rearrangement of the Breguet range equation, outlined in Ref. 18, a model of an assumed weight change profile for each aircraft can be developed. Let dW denote a change in weight of an aircraft due to fuel consumption over an increment of time dt, then given a thrust specific fuel consumption factor, C_t, and the thrust available T_A, the following relation holds

$$dW = -C_t T_A dt, \tag{3}$$

which rearranged with respect to dt is

$$dt = \frac{-dW}{C_t T_A}. \tag{4}$$

For the incremental distance dr travelled by the aircraft, over an increment of time dt, equation (4) is multiplied by a stream-free velocity V_∞ so

$$dr = V_\infty dt = \frac{-V_\infty dW}{C_t T_A}. \tag{5}$$

Rearranging equation (5) leads to the rate of fuel burnt per unit of distance

$$\frac{dW}{dr} = -\frac{C_t T_A}{V_\infty}. \tag{6}$$

Assuming steady level flight, then thrust available, T_A, should equal thrust required, T_R, and for a given coefficient of lift, C_L, and drag, C_D, then $T_A = T_R = \frac{W}{C_L/C_D}$ (which depends on W). The W required to evaluate this equation is determined after a certain flight distance, by following though with this derivation enables its calculation. First integrate equation (5) between the limits $s = 0$ (when $W = W_0$, the initial weight) and $s = R$ (when $W = W_1$, the final weight),

$$R = \int_0^R dr = \int_{W_0}^{W_1} \frac{V_\infty dW}{C_t T_A}, \tag{7}$$

$$R = \int_{W_1}^{W_0} \frac{V_\infty}{C_t} \frac{C_L}{C_D} \frac{dW}{W^{1/2}}. \tag{8}$$

Using the definition that for a given density ρ_∞, $V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$ results in

$$R = \int_{W_1}^{W_0} \sqrt{\frac{2}{\rho_\infty S}} \frac{C_L^{1/2}/C_D}{C_t} \frac{dW}{W^{1/2}}. \tag{9}$$

Assuming constant C_t, C_L, C_D and density ρ_∞ (at a constant altitude) then

$$R = \sqrt{\frac{2}{\rho_\infty S}} \frac{C_L^{1/2}/C_D}{C_t} \int_{W_1}^{W_0} \frac{dW}{W^{1/2}}, \tag{10}$$

$$R = \sqrt{\frac{2}{\rho_\infty S}} \frac{C_L^{1/2}/C_D}{C_t} \left(2W_0^{1/2} - 2W_1^{1/2}\right), \tag{11}$$

completing the derivation of the Breguet range equation.[18] Setting $M = \sqrt{\frac{2}{\rho_\infty S}} \frac{C_L^{1/2}/C_D}{C_t}$ to be the contribution of the constant terms then

$$R = M\left(2W_0^{1/2} - 2W_1^{1/2}\right), \tag{12}$$

$$W_1 = \left(\frac{M\sqrt{W_0} - R}{M}\right)^2 = \left(\sqrt{W_0} - \frac{R}{M}\right)^2. \tag{13}$$

M is assumed to be a non-zero constant, so given a distance R and initial weight W_0 an aircraft's weight can be calculated at that point. Equations (6) and (13) enable an estimation of fuel burn rates after any distance (up to the range of the aircraft). Equation (13) requires knowledge of an initial weight in order to estimate any en route weights, therefore it is necessary to also calculate the required fuel for the entire journey. The total initial fuel is defined to be the fuel required to fly the entire journey plus enough reserve fuel, this will be a large factor in the overall take off weight. In general, formations must deviate from their individual solo routes in order to meet up with other formation members, increasing the total distance travelled (even if they burn less fuel in doing so). Therefore in order for an aircraft to fly a formation route it must, at least as a conservative estimate, carry enough fuel so that it could, if necessary, fly it entirely solo without any reduction in fuel burn from formation flight. In general this means that any aircraft planning to join in formation must carry more fuel relative to the same aircraft flying solo and in turn it will burn fuel at a slightly increased rate. As there are currently no rules in place for commercial formation flight to address this, an assumption is made that for either solo or formation flight each aircraft must carry enough fuel to take off, land and fly 110% of the full cruise distance. In the absence of specific aircraft payloads this paper assumes a nominal Zero Fuel Take Off Weight (ZFTOW), taken directly from BADA, to which the weight of fuel required is then added to reach an estimate for W_0. This assumption means that the initial take off weight is really just a function of cruise distance so can be incorporated into the weight equation.

Algorithm 1 An Outline of an Iterative Approach to Optimal Routes with Varying Fuel Weights

$w_{A,N} \leftarrow w^*_{A,N}$; $w_{B,N} \leftarrow w^*_{B,N}$; ▷ Create initial weightings based on nominal fuel burn
$w_{fN} \leftarrow \alpha(w_{A,N} + w_{B,N})$ ▷ For rendezvous and break point $N \in \{1, 2\}$
$W_{A,0} \leftarrow$ MTOW of Aircraft A ▷ Set aircraft A weight with maximum fuel
$W_{B,0} \leftarrow$ MTOW of Aircraft B ▷ Set aircraft B weight with maximum fuel
while $w_{i,N}$ Not Converged **do** ▷ While the difference is too large
  S $\leftarrow$ OptimalRoute(RouteA,RouteB,$w_{A,N}$,$w_{B,N}$,$w_{f,N}$) ▷ Find optimal route
  for `Each Aircraft` $F_i \in \{F_A, F_B\}$ **do** ▷ $i \in \{A, B\}$
    for `Each Point` $P_{i,N}$ **do** ▷ For each rendezvous and break away point
      $dP_{i,N} \leftarrow$ distance($P_{i,N}$) ▷ Calculate distance to reach each point
      $w_{i,N} \leftarrow \frac{dW}{dr}\|_{P_{i,N}}$ ▷ Update fuel burn rate at P_N
    end for
    D(p_a, p_b, p_c)$\leftarrow$ distances(S, F_i) ▷ Calculate distances for each section of flight
    $W_{i,1} \leftarrow \left(\sqrt{W_{i,0}} - \frac{\sum D(p_a,p_b,p_c)}{M}\right)^2$ ▷ Calculate and update final weight
    $W_{i,0} \leftarrow \left(\sqrt{W_{i,1}} + \frac{\sum D(p_a,p_b,p_c)}{M}\right)^2$ ▷ Calculate and update initial weight
    $Cost_i \leftarrow W_{i,0} - W_{i,1}$ ▷ Get cost for the flight
  end for
end while
return S, C_A and C_B ▷ Return the final solution and costs

As detailed in Algorithm 1 a nominal fuel burn rate is first used for an estimated initial solution, from which W_0 and the geometric weights are adjusted based on the distance needed to fly to reach the given rendezvous point (and analogously between the break away point and the destination). Repeating this process two or three times quickly converges to a solution which is optimal (to a sufficient degree of accuracy) for varying fuel weights. This outlines a method for optimal routing for weights as a function of distance, a comparison of the results of this, along with the nominal fuel burn rates, for a transatlantic case study are presented in section VI.

V. The Impact of Wind

In the presence of wind aircraft will commonly deviate from a great circle path. Aircraft usually attempt to fly the path of least time avoiding areas of large headwinds, opting for those with less resistance or even tailwinds. Flying a large portion of a flight with a reasonable tailwind, such as a east bound transatlantic flight, can reduce flight times by as much as a few hours compared to their west bound counterparts.[22] Due to this, North Atlantic Tracks (NATs) are published daily, both to alleviate traffic and to account for this portion of predictability in certain weather patterns. Our previously discussed geometric method does not, however, take into account the almost certain likelihood of encountering wind. It is therefore beneficial to see how well an optimal assignment, based on geometric formations, performs in the presence of predictable wind. The method outlined is not intended to be a likely candidate for routing on a global scale, rather it is designed to benchmark the fast geometric approach for the macro scale.

This paper does not attempt to model altitude changes, instead a flight level is fixed and a single 'layer' of predicted wind is chosen. A randomly generated set of points are assigned vectors with a random magnitude and direction, an interpolation is then applied between points to create a smooth wind field. In the absence of an analytic solution for routing optimally, in the presence of wind, a numerical approach is used with a similar notion of weighting (that is, during the formation section of the flight a fixed proportionality constant to represent an assumed formation flight fuel saving is used). Then for each section of each flight a finite number of variable way-points are generated, filling in the gaps using a series of interpolated great circle points. The route sections are constrained to share either the common rendezvous or break away point in such a way as to simulate a formation routing. At each interpolated point the dot product of the direction of the route with the direction of the wind is computed generating the contribution of the head or tailwind V_W. This is then incorporated into the cost function (which is a function of the distance d) using an identical proportional discounting factor for the section of the flight flown in formation. The wind cost for each incremental step dx is therefore, for a given aircraft cruise velocity V_∞, a summation of all incremental costs

$$dx_W = \frac{dx}{(1 + (V_W/V_\infty))}. \tag{14}$$

An 'active-set' optimization within Matlab's 'fmincon' function, is used to determine the values of the variable way-points for every possible pairing. This takes around 30 seconds to find a result per formation pair. However, as this process is highly parallelizable (simply enumerating all combinations) means the process can easily be split up and run on a cluster of computers. Although as discussed in section II, the nature of the problem means that as the formation size (or number of routes to consider) grows the number of possible combinations needing to be evaluated drastically increases. Therefore an unrefined and relatively slow method of optimizing routes in the presence of wind would not be suitable for total enumeration for a problem much larger than the case study discussed in section VI. It is, however, better suited to a 'post-process' optimization, whereby a smaller subset of geometrically optimized pairs are re-optimized to include the effects of wind. The next section outlines the results of such a scenario.

VI. Comparison of Methods: A Transatlantic Case Study

Given an example set of data for 210 common transatlantic flights between 26 US and 42 European airports. The aim is to create formations of size two in order to minimize the total cost (kg of fuel burnt) of the entire fleet. Each flight is treated as non-greedy, doing what is best for the fleet as a whole rather than individual gain. In this sense the fleet could be thought to represent a single airline company. The approach taken for each of the methods previously outlined (sections II - V) is to first evaluate all possible ways (21945) of making pairs of two from 210 flights using a fixed proportional discounting rate of 0.9 (from Table 1) for the formation section of the flight. As each flight can only be in one formation (or fly solo) Gurobi's[23] MILP (Mixed Integer Linear Program) solver is used to generate an optimal subset of pairs to be the final solution. Solving in such a way is highly effective for smaller problems as a MILP is NP-hard under certain conditions, such as number of variables, number of constraints and the convexity of the problem.[24] The non-convex nature of this problem, i.e. there are many possible local minimum, means that finding a global minimum is already a difficult task. Therefore by increasing the size of the problem (the number of variables) the amount of resources needed to solve it will also increase.

An alternative approach for much larger problems (a route list larger than a few thousand) would be to use a heuristic algorithm such as Simulated Annealing[25] (whereby a stochastic rule picks a new solution state and then probabilistically decides whether or not to keep it). The time taken to run this algorithm changes linearly with the number of iterations, so it can be useful for finding a solution in a fixed amount of time. As this method is a random process there is no guarantee that the solution will converge to the global optimum. Therefore it is useful either to find a 'good' solution in finite time or at least give a lower bound on a possible global percentage saving. The relatively small size of this particular case study, however, is suited to a MILP. Each of the cases outlined below are the same size of problem (although the convexity may change slightly) and therefore the runtime for the MILP is consistently around 2 seconds.

A. Geometric Solutions With Nominal or Differential Weights

The geometric method in its simpler form is a very fast way of finding solutions, it is possible to enumerate around half a million optimal routes in less than a minute. This makes it a likely candidate for larger problems. To compute every combination of formations for this relatively small problem takes just over 2 seconds for nominal weighting (using the estimates outlined in BADA) and 4 seconds for differential weighting (using the iterative Breguet range equation). Aircraft specific weights (as section III explores), based on predefined nominal fuel burn rates per km of cruise, result in a global saving of around 8.593%. However, as outlined in section III the rate at which an aircraft burns fuel (in constant level flight) is dependent on the distance already travelled. Instead of a constant nominal fuel burn weighting, the weights now vary as each aircraft traverses its flight path. This results in a fractionally reduced saving of around 8.522%, however this is to be expected as the level of detail in the problem is increased and can be attributed in part to having to carry a greater amount of weight to allow for reserve fuel. In a similar scenario but with identically nominally-weighted aircraft the potential for fuel saving is based solely on the geography of the routes, that is, the location of the departure and destination points, was found to be 8.643%.[17] The introduction of aircraft specific weightings means that it is not only the location of routes but also the aircraft that fly them which impact potential savings. It is however, by removing assumptions from the problem statement which allows for a tighter upper bound on the potential fuel savings attainable from realistic flight.

Calculation of Weight	Total Enumeration Time	Fuel burn Saving
Nominal	4.128s	8.593%
Differential	6.422s	8.522%

Table 2. Fuel Saving Percentages Against Their Respective Solo Routes for Different Wind Fields

B. Geometric Solutions For Wind-Optimal Routes

For this case study a wind field is created to be representational of the jet stream over the Atlantic (see Figure 2). The method used to enumerate each pairing is slow, meaning in order to get results in a reasonable time the enumeration stage is split into a number of smaller subproblems run on a computer cluster (results of this paper are based on using the University of Bristol's High Performance Computer (HPC) BlueCrystal Phase 2[26]). The total wall time for this is around 150 hours, drastically more than a geometric wind-free solution. However this method is more a proof of concept to benchmark the geometric results. Let GOPW stand for the Geometrically Optimal solution Pairings which are then optimized for Wind, WOP stand for the Wind Optimal solution Pairings and WOS for Wind Optimal Solo routes.

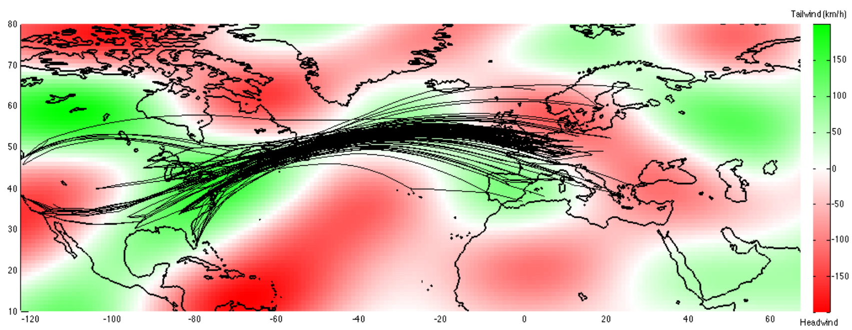


Figure 2. Eastbound Wind-Optimal Formation Pairings (WOP)

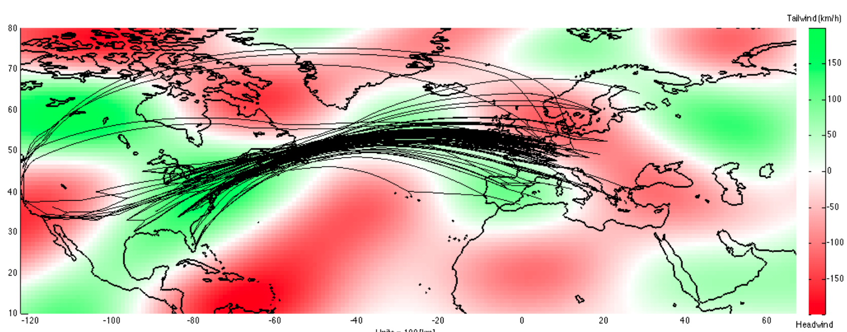


Figure 3. Eastbound Geometric Estimate to Wind-Optimal Formation Pairings (GOPW)

The WOPs result in potential savings, against WOS routes, of 9.316%. A proportion of this saving comes from cruising with a tailwind (as visible in Figure 2) so makes any possible saving dependent on the particular wind field used. The 'wind-distance' travelled (i.e. the equivalanet km cost to fly the same distance without wind) is 1.016% less than the solo routes while the actual distance flown is about 0.511% less for formations. The solo routes appear to need more freedom to find a better path through wind as their deviations are not met by formation fuel savings. This can be somewhat attributed to the nature of the wind field, whereby the majority of the tailwinds are in the mid north-atlantic, so formations are already likely to fly close to these benificial regions.

A promising result arises when taking the solution pairings from the geometrically optimal solution (which ignores potential wind) and then optimally route them through the same wind field (GOPW as in Figure 3). The total cost is around 9.094% less than their WOS counterparts, which is under half a percentile worse than the solution for WOP. This implies is that it may not be necessary to evaluate every wind-optimal route combination, but rather leave it as a post-process. That is, (1) Use the fast geometric method to enumerate all possible pairings (ignoring wind), (2) Run a MILP (or similar) to create a subset of optimal formation pairs and finally (3) Optimize the subset for a predicted wind field. Moreover one can see immediately from Figure 3 that three of the geometric pairings may not be best suited to this particular wind field (the ones which travel over Greenland). It may therefore be beneficial to consider taking the six underlyingsolo routes and look at possible neighbouring solution pairs for further improvement.

Examining the results for the same set of flights for the same wind field but travelling west bound. The resulting pairings for the WOP saves around 9.325% against 8.181% for the GOPW. The difference is a little higher, around the 1% mark, adding to the idea that the geometric solution pairs may be a reasonable estimate for the global wind solution. Figures 4 and 5 show how the routes clearly avoid the green areas (headwinds when travelling westerly) and take a route either above or below. The formation routes taken are decreased by 0.817% and 0.156% for wind-distance and absolute-distance respectively compared to the solo routes.

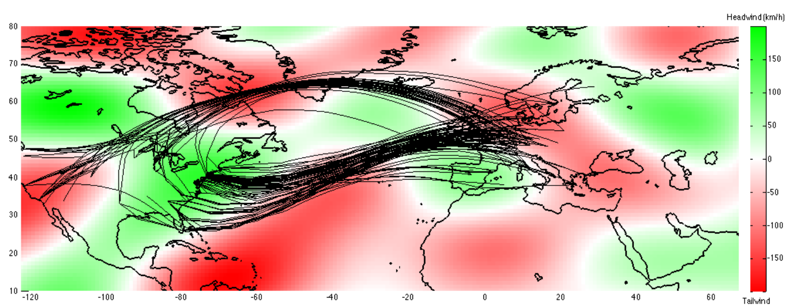


Figure 4. Westbound Wind-Optimal Formation Pairings (WOP)

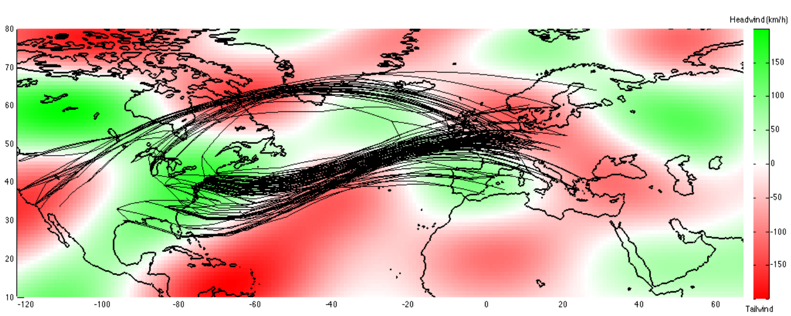


Figure 5. Westbound Geometric Estimates to Wind-Optimal Formation Pairings (GOPW)

Finally consider a much more volatile wind field where there are a greater number of distinct areas of differing winds with a larger difference in peak values. The results show a 9.343% saving for WOP and a 8.224% saving for the GOPW estimates against WOS. With respective decrease of 2.292% and 1.231% to wind-distance and absolute-distance. It is clear from Figures 6 and 7 that the routes passing through the more volatile wind field returns a less smooth path, jumping between areas of less resistance.

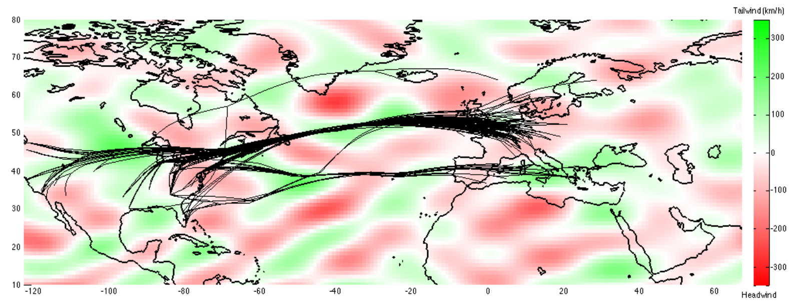


Figure 6. Wind-Optimal Formation Pairings (WOP) Through a Volatile Wind Field

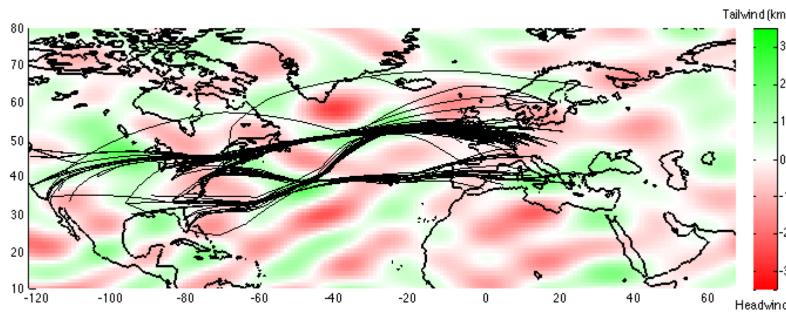


Figure 7. Geometric Estimated Formation Pairings (GOPW) Through a Volatile Wind Field

	WOP		GOPW		
Wind Field	Runtime	Saving (%)	Runtime	Saving (%)	Percentile difference
Jet stream (East)	150 hrs	9.316	43m	9.094	0.223
Jet stream (West)	150 hrs	9.325	43m	8.181	1.144
Volatile	150 hrs	9.343	43m	8.224	1.119

Table 3. Comparison of Fuel Saving Percentages Against Respective Solo Routes of WOP and GOPW

Now the actual percentage savings are more heavily influenced by the particular winds encountered. The amount of deviation, between savings for wind-optimal and geometrically-optimal formation assignment are more important. Although it may not attain a global optimum for a more realistic model, it allows the use of the fast geometric approximation to estimate solutions to much larger problems in realistic time-frames (some 200 times faster). Once a smaller solution set is determined it can be post-processed to further improve the route to account for predictable wind patterns.

VII. Conclusion

This paper has explored two distinct methods for finding optimal routes for formation flight. Firstly an extension to the Fermat-Torricelli problem allows the decoupling of a complex problem, providing a fast and effective framework to find optimal formations for a list of routes. Using a set of general aircraft performance coefficients allows a more accurate representation of routes containing distinct aircraft to be incorporated into the solution. The introduction of either a nominal or differential aircraft weighting scheme allows formation fleets to be more accurately assigned and routed to account for differing aircraft efficiencies. The simple iteratively-updating scheme also allows room for possible expansion in future, such as a more accurate calculation of the specific proportionality discount factor between particular aircraft pairings.

Secondly the modelling of wind fields to represent how aircraft might route in the presence of significant weather, while increasing the amount of detail in the problem to create tighter bounds on the possible optimum. The geometric method provides an 'ideal-world' solution, whereby all flight paths are great circle and do not encounter any kind of weather. The results in section V and Table 3 show that the geometric method acts as a reasonable estimate for the formation fleet assignment problem in the presence of wind. Moreover as the GOPW solutions only provide an initial estimate, it would be of interest to explore possible methods to improve upon it, such as a heuristic algorithm which prioritizes minimizing the number of times a wind optimal route is calculated.

The unrefined nature of the algorithm used to compute optimal wind paths means that it is not a likely candidate for assessing much larger problems. It would therefore also be beneficial to look at either refining it or using other methods, such as particle swarm optimization, to improve runtime, hopefully enabling the assessment of much larger problems (and in a greater depth, including routing through wind for formation fleets of more than two aircraft). Lastly while this paper does not attempt to assess the impact of a dynamic model it would be of interest to see how aspects of timing and uncertainty affect possible routings, in particular how to route for both predictable and unpredictable dynamic wind fields, however it is left for future work.

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